4.6 Bandwidth-Efficient Modulations

4.79. We are now going to define a quantity called the "bandwidth" of a signal. Unfortunately, in practice, there isn't just one definition of bandwidth.

$f_2 - f_1$

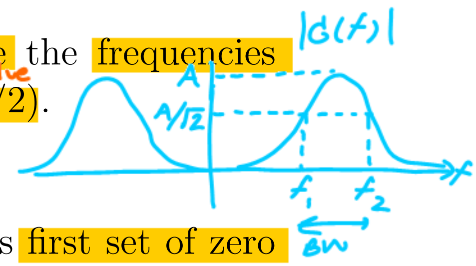
Definition 4.80. The **bandwidth** (BW) of a signal is usually calculated from the differences between two frequencies (called the bandwidth limits). Let's consider the following definitions of bandwidth for real-valued signals [3, p 173]

(a) **Absolute bandwidth**: Use the highest frequency (f_{max}) and the lowest frequency (f_{min}) in the positive-f part of the signal's nonzero magnitude spectrum.

- This uses the frequency range where 100% of the energy is confined.
- We can speak of absolute bandwidth if we have ideal filters and unlimited time signals.

$10\log_{10} x = 3$, $x = 10^{0.3} \approx 2$

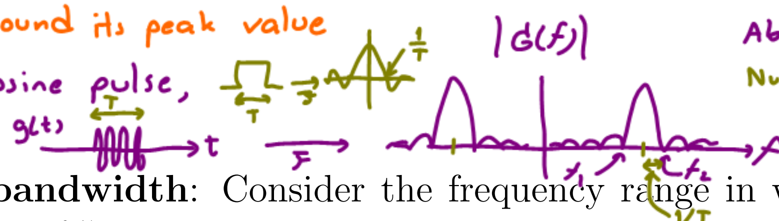
(b) **3-dB bandwidth** (**half-power bandwidth**): Use the frequencies where the signal power starts to decrease (around its peak value) by 3 dB (1/2).



- The magnitude is reduced by a factor of $1/\sqrt{2}$.

(c) **Null-to-null bandwidth**: Use the signal spectrum's first set of zero crossings. around its peak value

Ex. For cosine pulse,



Absolute BW $= \infty - 0 = \infty$

Null-to-null BW $= 2 \times \frac{1}{T}$

$= \frac{2}{\text{pulse width}}$

(d) **Occupied bandwidth**: Consider the frequency range in which $X\%$ (for example, 99%) of the energy is contained in the signal's bandwidth.

(e) **Relative power spectrum bandwidth**: the level of power outside the bandwidth limits is reduced to some value relative to its maximum level.

generalized idea

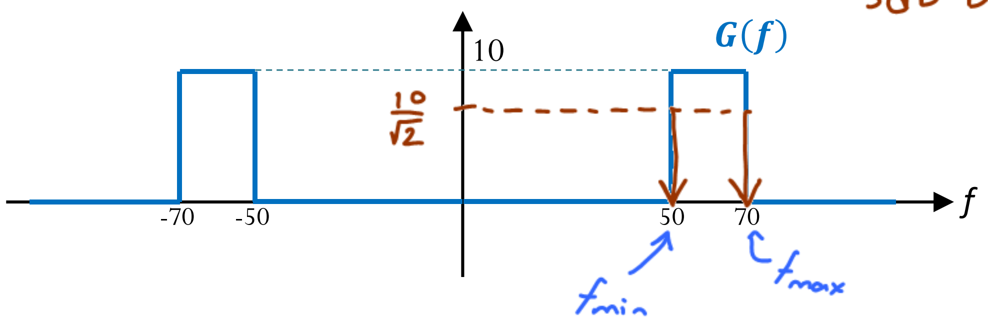
- Usually specified in negative decibels (dB).
- For example, consider a 200-kHz-BW broadcast signal with a maximum carrier power of 1000 watts and relative power spectrum bandwidth of -40 dB (i.e., 1/10,000). We would expect the station's power emission to not exceed 0.1 W outside of $f_c \pm 100$ kHz.

Example 4.81.

Absolute BW = 70 − 50 = 20

Null-to-null BW = 70 − 50 = 20

3dB BW = 70 − 50 = 20

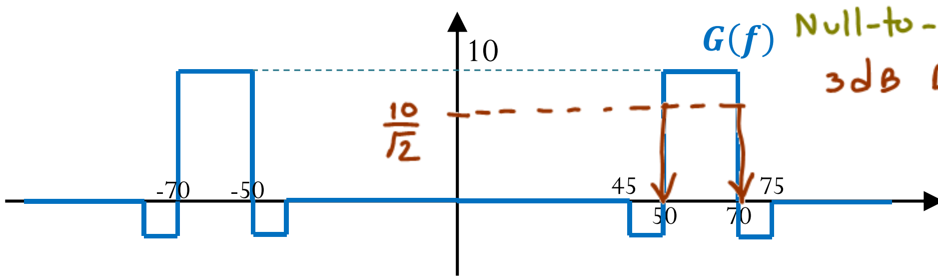


Example 4.82.

Absolute BW = 75 − 45 = 30

Null-to-null BW = 70 − 50 = 20

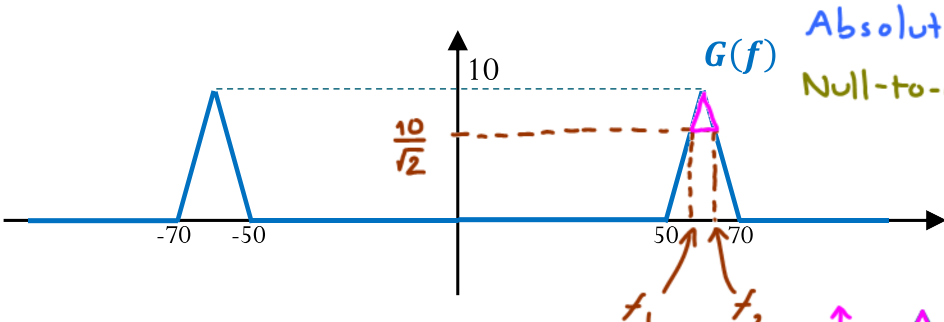
3dB BW = 70 − 50 = 20

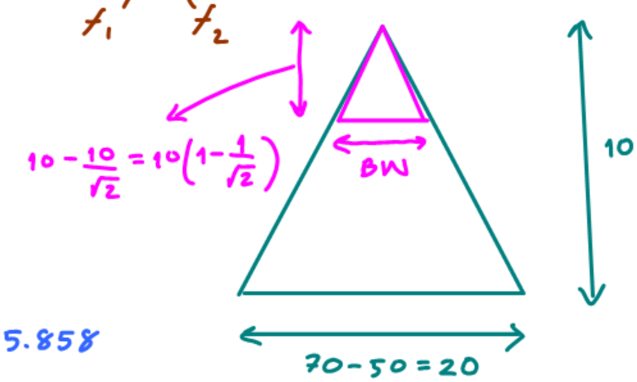


Example 4.83.

Absolute BW = 70 − 50 = 20

Null-to-null BW = 70 − 50 = 20





$$\frac{BW}{10\left(1-\frac{1}{\sqrt{2}}\right)} = \frac{2\cancel{0}}{1\cancel{0}}$$

$$\text{3dB BW} = 2 \times 10\left(1 - \frac{1}{\sqrt{2}}\right) \approx 5.858$$

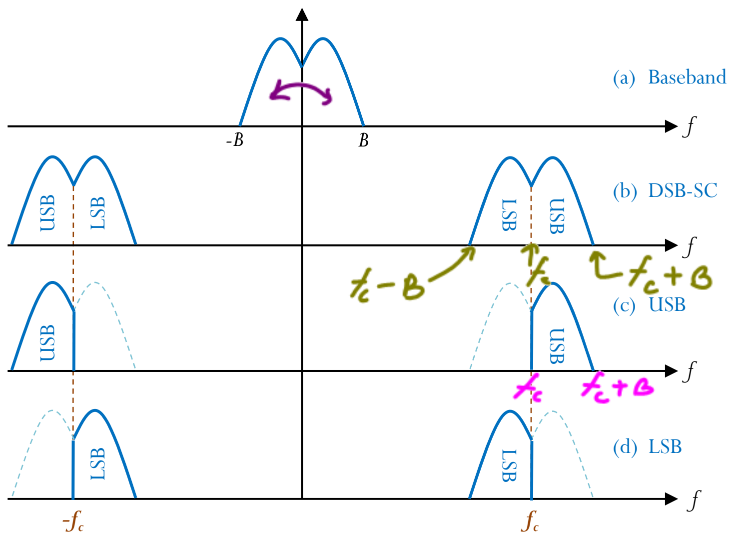
CH3 : DSB-SC

transmitted signal
modulated signal

Example 4.84. Message bandwidth and the transmitted signal bandwidth

$|M(f)|$

$|X(f)|$



Absolute BW $= f_{max} - f_{min} = B - 0 = B$

Absolute BW $= f_{max} - f_{min} = (f_c + B) - (f_c - B) = 2B$

Absolute BW $= f_{max} - f_{min} = (f_c + B) - f_c = B$

Figure 31: SSB spectra from suppressing one DSB sideband.

4.85. BW Inefficiency in DSB-SC system: Recall that for real-valued baseband signal $m(t)$, the conjugate symmetry property from 2.30 says that

$$M(-f) = (M(f))^* .$$

The DSB spectrum has two sidebands: the upper sideband (USB) and the lower sideband (LSB), each containing complete information about the baseband signal $m(t)$. As a result, DSB signals occupy twice the bandwidth required for the baseband.

4.86. Rough Approximation: If $g_1(t)$ and $g_2(t)$ have bandwidths B_1 and B_2 Hz, respectively, the bandwidth of $g_1(t)g_2(t)$ is $B_1 + B_2$ Hz.

This result follows from the application of the width property[18] of convolution[19] to the convolution-in-frequency property.

Consequently, if the bandwidth of $g(t)$ is B Hz, then the bandwidth of $g^2(t)$ is $2B$ Hz, and the bandwidth of $g^n(t)$ is nB Hz. We mentioned this property in 2.42.

[18]This property states that the width of $x * y$ is the sum of the widths of x and y.

[19]The width property of convolution does not hold in some pathological cases. See [5, p 98].

4.87. To improve the spectral efficiency of amplitude modulation, there exist two basic schemes to either utilize or remove the spectral redundancy:

(a) Single-sideband (SSB) modulation, which removes either the LSB or the USB so that for one message signal $m(t)$, there is only a bandwidth of B Hz.

(b) Quadrature amplitude modulation (QAM), which utilizes spectral redundancy by sending two messages over the same bandwidth of $2B$ Hz.